Prediction of Hypersonic Laminar Boundary-Layer/Shock-Wave Interactions

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Theme

NUMERICAL solution of the compressible time-dependent Navier-Stokes equations is one method of obtaining the theoretical solution of the interaction of an oblique shock wave and a laminar boundary layer. MacCormack[1] and MacCormack and Baldwin[2] used the method of MacCormack[3] to generate solutions for low supersonic Mach numbers. The purpose of this Synoptic is to ascertain the validity of the MacCormack algorithm for laminar boundary-layer/shock-wave interactions in the hypersonic regime. Results of several numerical solutions were compared with experimental data from the von Karman Facility (VKF) Hypersonic Wind Tunnel (B) and the Langley Research Center (LRC) Mach 8 Variable Density Tunnel. Agreement between the numerical solution and the data was generally satisfactory, although the extent of separation was underpredicted in interactions with large separated regions. When applied to hypersonic interactions having large separated and recirculating regions, the method gave marginal performance. Lack of spatial resolution in the longitudinal direction is the suspected cause of the discrepancies.

Contents

The MacCormack method, which is an explicit, split, two-step algorithm, was used to solve the laminar, compressible, time-dependent Navier-Stokes equations expressed in conservative form. The equations, together with the details of the algorithm, are given by MacCormack.[2] Because of the difficulty in maintaining laminar flow throughout hypersonic boundary-layer/shock-wave interactions for strong shock waves and moderate-to-high Reynolds numbers, the incident shock angles (Fig. 1 for schematic) in this study are limited to a maximum of 16 deg and the Reynolds numbers to a maximum of 10^6/ft.

The computed pressure ratio at the wall, p_w/p_∞, is compared with measured data of an AEDC-VKF Tunnel B experiment in Fig. 1.[4] Agreement is excellent particularly near the peak pressure location where the correct shape is generated, and in the midrange where the correct pressure gradient is obtained. No evidence of separation was obvious from the data, and no separated region was predicted by the program.

An additional hypersonic laminar boundary-layer/shock-wave interaction was attempted using as a basis the data of Kaufman and Johnson[5] taken in the LRC Mach 8 Variable Density Tunnel. The results of the computation, as well as the relevant experimental data, are presented in Fig. 2. Agreement is satisfactory particularly near the peak pressure location, and the predicted pressure gradient within the interaction region agrees well with the experimental data. As in the case of Fig. 1, no separated region is evidenced by the data or predicted by the program. The major discrepancy occurs near the beginning of the interaction region where the experimental pressure is about 50% higher than the computed pressure. The wall temperature which can strongly affect the viscous interaction was taken as 0.5 of the total temperature (as per the suggestion of Kaufman and Johnson). The strong interaction analysis of Bertram and Blackstock[6] (which agrees favorably with a wide range of experimental data) was examined for the conditions of Fig. 2 and yielded essentially the same viscous-induced pressure as predicted by the MacCormack algorithm. This increases confidence in the ability of the code to realistically predict leading-edge viscous interaction but does little to explain the pressure discrepancy in Fig. 2. Nevertheless, the laminar boundary-layer/shock-wave interaction is predicted satisfactorily.

Additionally, at the hypersonic conditions used in this Synoptic, it was necessary to add damping terms to stabilize the solution. These terms, especially in regions of large gradients, could cause "smearing" of the numerical solution.

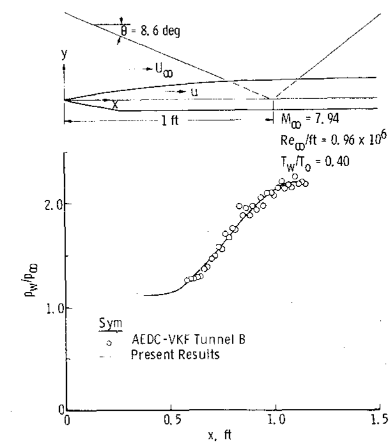

Fig. 1 Laminar hypersonic boundary-layer/shock-wave interaction using AEDC-VKF Tunnel B conditions.

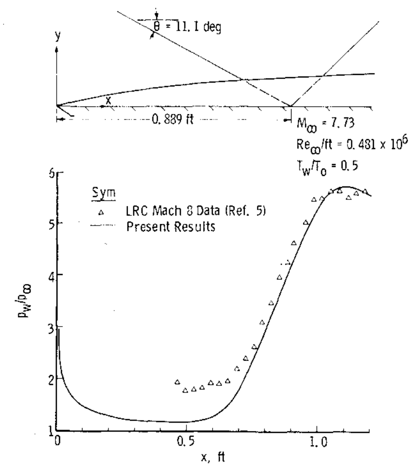

Fig. 2 Laminar hypersonic boundary-layer/shock-wave interaction using LRC Mach 8 Tunnel conditions.

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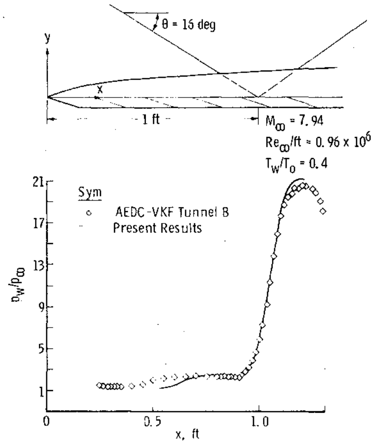

Fig. 3 Laminar hypersonic boundary-layer/shock-wave interaction with separated region.

The last interaction examined is based on AEDC-VKF Tunnel B conditions and has a large separated region. The tunnel conditions used and the results of a time-dependent numerical solution are given in Fig. 3. This solution was generated by computing a hypersonic flat-plate solution and using this solution as initial conditions for the upstream portion of the computational region. Agreement between computed and experimental data was adequate in the region about the peak pressure and good for the pressure gradient near the end of the interaction. The middle portion of the interaction exhibits the correct plateau of pressure. The largest area of disagreement is the beginning of the interaction where the computed plateau and separation regions are much shorter than those indicated by experimental data. Hung and MacCormack[7] reported similar problems in predicting separated flows for hypersonic compression ramps with large regions of separated flow. Lack of spatial resolution is a possible cause of this anomaly.

The cell Reynolds number as an index of spatial resolution has previously been examined by MacCormack[1] who suggested a cell Reynolds number on the order of two for each coordinate mesh spacing if every term of the Navier-Stokes equations is to receive adequate support. For this particular case, a cell Reynolds number based on Δy within the viscous-dominated region has a typical value of 6.0; i.e.,

$$Re_{\Delta y} = \rho v \Delta y / \mu \approx 6.0$$

and the cell Reynolds number based on Δx within the viscous-dominated region has a typical value of 1250; i.e.,

$$Re_{\Delta x} = \rho u \Delta x / \mu \approx 1250$$

Thus, the y-mesh should adequately support all terms, whereas the x-mesh would not adequately support all terms. MacCormack[1] suggests that the inadequately supported terms are not necessary for boundary-layer flow calculation, but anomalies between the experimental data and the numerical solution show quantitative differences, which could be the result of inadequate mesh resolution in the x-direction. If the grid resolution in the x-direction is the cause of the anomalous behavior for the large region of separation, then the needed reduction of several orders of magnitude in Δx would increase CPU time and core storage to untenable levels. The above discussion is valid not only for the MacCormack algorithm but for all other discrete spatially elliptical methods.

The examples illustrated in Figs. 1 and 3 have essentially the same cell Reynolds numbers based on Δx. The example presented in Fig. 1, however, possesses no region of separation while the example presented in Fig. 3 has a large separated region. This suggests that the ability of the x-grid to support *all* terms of the Navier-Stokes equations adequately near the separation point is of particular importance. The good agreement between the computed results and the experimental data about the region of steep pressure gradient ($x < \sim 1$) in Fig. 3 indicates that the grid solution at reattachment is less critical than the grid resolution at separation.

Acknowledgments

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